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An analysis of experimental results on fluctuations of muon and electron numbers in extensive air showers

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Abstract. The theoretical results derived in the previous paper by de Beer *et al.* in 1968 have been compared with experimental observations, with reference to the fluctuations in the total numbers of muons and electrons and to the variation of the mean number of muons with shower size.

It is found that the derivation of accurate parameters from experimental data is very difficult in view of the presence of a variety of bias effects. However, good agreement appears, in general, when comparison is made of the predictions of our preferred model (which assumes a CKP type of energy distribution for the secondary pions from high-energy interactions).

The nature of the mass composition of the primaries is still unclear in view of the rather low sensitivity of most parameters to the mass spectrum. What evidence there is, from the (N_μ, N_e) relation, gives only a very slight indication of a change in the primary composition above 10^{15} ev.

1. Introduction

The preceding paper by de Beer *et al.* (1968, to be referred to as II) describes the results of theoretical studies of near-vertical extensive air showers, with particular reference to the fluctuations in the electron and muon components. In the present paper comparisons are made between the predictions of II and the experimental results reported by a number of workers. The object of the work is to throw light on two aspects of the subject of the cosmic radiation: the mass spectrum of the primary particles, and the characteristics of the interactions of nuclear-active particles with air nuclei at very high energies.

The experimental results chosen concern the fluctuations in the electron and muon components, together with the variation with size of the mean number of muons, all the measurements referring to near-vertical showers at or near sea level.

2. Fluctuations in muon and electron number

2.1. *Experimental uncertainties*

A variety of experimental uncertainties combine to make the comparison of the theoretical predictions with the experimental data a difficult procedure. Considering the widths of the distributions of N_e (electron size) and N_μ (muon number), we find that the most serious uncertainties are statistical errors in N_μ arising from the limited areas of muon detectors commonly used, errors in N_μ/N_e due to the lack of accurate knowledge of shower direction and errors in core location which affect both N_μ and N_e. The distributions have been shown to be sensitive to the region of radial distance in which muon density measurements are made, so that, where this region is not known, a further uncertainty is present.

In the comparisons corrections have been estimated to allow for the above effects, where these have not been made by the original authors, and further factors have been applied to standardize the results to a mean angle of shower incidence of 0^0.

2.2. *The distribution in N_e for fixed N_μ*

Comparison has been made with the results of two groups: Tokyo (Hasegawa *et al.* 1963, Ogita 1962) and Moscow (Khristiansen 1967, private communication, Vernov *et al.*

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1967). A comparison is made with expectation in figure 1, where the predicted curves refer to the hypothetical case where the measurements give the total number of muons (and electrons), derived from very many detectors spread over the shower front. The Tokyo results have been corrected by the present authors to allow for the effects mentioned in § 2.1. In so far as the measurements were made at small distances from the core, the correction to allow for correlation between muon density and distance of the muon detector from the shower core is large (an increase in $\sigma/\bar{N}_e \sim 2$).

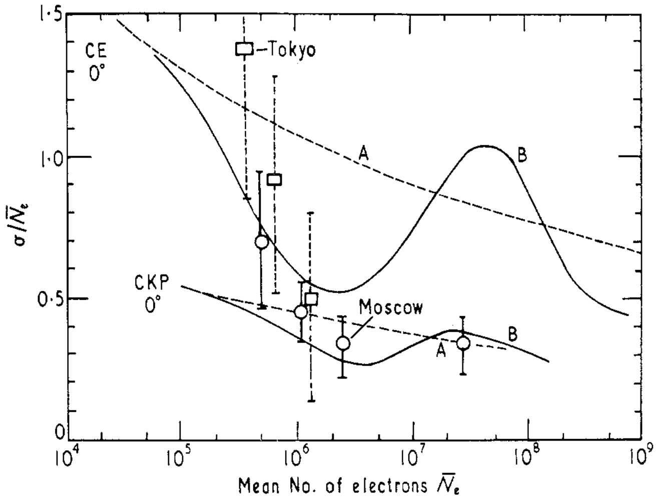

Figure 1. Relative standard deviation of N_e for fixed N_μ. The theoretical curves refer to showers incident in the vertical direction; A and B correspond to alternative assumptions about the nature of the primary composition—unmodulated and modulated, respectively.

Only corrections for the radial correlation and standardization to $\theta = 0°$ are necessary for the Moscow points since the authors themselves have corrected for the other effects. The correlation corrections are close to unity in view of the radial distances being comparatively large in this experiment, and allowance for the mean shower angle of 20°, to convert to what would have been recorded for a mean angle of 0°, amounts only to a reduction in $\sigma/\bar{N}_e$ by 12%.

A comparison of the corrected results with expectation shows immediately that, assuming the version of the model using the CKP relation, it is not possible to distinguish between the alternative primary compositions. At first sight the values for small sizes give support for the CE rather than the CKP model, but such an indication is not regarded as conclusive. This is because the high value found in the Tokyo experiment, which contributes largely to the indication, arises from the large contribution to $\sigma/\bar{N}_e$ from a very small number of showers with large values of N_e. Furthermore, the corrected value of $\sigma/\bar{N}_e$ is sensitive to the magnitude of the correction, subtracted in quadrature from the measured width, which allows for errors in N_μ, N_e, etc., this correction being inevitably largest at small values of $\bar{N}_e$.

At higher values of N_e, where the results and their corrections are expected to be more reliable, it is seen that there is support for the CKP model but, as was pointed out above, the results are too imprecise to enable a further distinction to be made between the alternative primary compositions.

2.3. *The distribution in N_μ for fixed N_e*

Many experiments have been carried out in which this distribution has been measured, but only those results for which the correction factor can be calculated with some confidence have been used. The theoretical curves, which refer to vertical showers, and the corrected points are given in figure 2. The point marked 'Lodz' comes from the work of Gawin *et al.*

(1967) and is based on measurements of some 100 000 showers. A comprehensive analysis of the experimental errors was made by these workers, and the appropriate reduction in $\sigma/\bar{N}_\mu$ was made by them; the only further correction applied by the present authors has been a small reduction to convert from the actual mean zenith angle of about 20° to 0°.

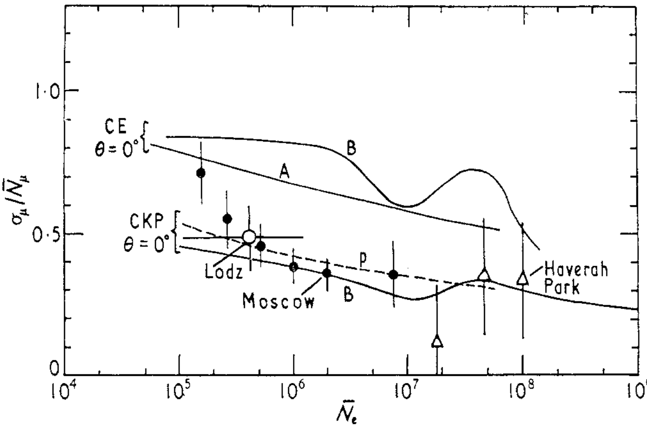

Figure 2. Relative standard deviation of N_μ for fixed N_e. The theoretical curves refer to showers incident in the vertical direction; A and B correspond to alternative assumptions about the nature of the primary composition—unmodulated and modulated, respectively—and p denotes protons.

The 'Moscow' points refer to measurements reported by Vernov *et al.* (1967) based on some 1100 showers. A number of corrections were applied by the original authors and the only further corrections necessary were to standardize the results to $\theta = 0°$ and to allow for the radial distance correlation referred to in II and § 2.1. The errors shown are those quoted by the original authors; in the light of our experience of the uncertainties in the various correction factors it is possible that they are somewhat underestimated.

Finally, data from the Haverah Park experiment (Pidcock 1967) have been analysed by the present authors and the values obtained are given in figure 2. The large errors shown result from the uncertainty in the corrections applied to the data for the various effects mentioned in § 2.1.

3. Lateral distribution of muons

A detailed comparison was made by de Beer *et al.* (1966) of the experimental results prior to 1966 with the lateral distributions expected for variants of the adopted model. It was shown there that the experimental points for high muon threshold energies and large distances from the shower axis were significantly higher than predicted. Later results from the Haverah Park experiment, by Earnshaw *et al.* (1967 a, b), have shown that the discrepancy increases in magnitude with increasing E_μ and r. A likely explanation is that the mean transverse momentum of the parents of the muons is higher than commonly assumed, although other changes in the character of high-energy interactions cannot be ruled out (see the discussion of the interpretation of the results by the present authors (de Beer *et al.* 1967)).

4. Variation of the N/N_e ratio with shower size

4.1. *Experimental uncertainties*

Data from a variety of experiments were collected in I and compared with expectation, and it was concluded that if the adopted model was correct in every particular then there was evidence for a significant flux of heavy particles in the primary radiation (the effective mean value of A was found to be about 4). The analysis was confined to showers having sizes in the region of 10^6 particles, and the conclusion was therefore applicable only to a restricted primary energy region.

In the present work an examination is made of the variation of the N_μ/N_e ratio with shower size, from which it is possible, in principle, to examine the variation of primary composition with energy. The derivation of an accurate value for this ratio from the experimental data is again a procedure of some difficulty on account of various errors and uncertainties. Because some systematic errors vary only slightly with mean size, attention has been confined to the variation with mean shower size $\bar{N}_e$ of the exponent α in the relation $\bar{N}_\mu \propto \bar{N}_e{}^\alpha$. One source of uncertainty in the value of α is the lack of precise knowledge of the forms of the muon and electron lateral distributions and their possible variation with shower size. Another uncertainty concerns the manner in which the mean number of muons is derived from a set of measured values of N_μ for showers with N_e lying in a narrow range. It appears customary to take a linear mean, but in fact the appropriate value is nearer the logarithmic mean, in view of the fact that a large part of the spread in N_μ values is due to errors (as described in § 2.1) which are distributed almost symmetrically on a logarithmic plot (in practice the median is nearly proportional to the logarithmic mean). Since the measured width of the distribution of N_μ falls with increasing shower size there is a significant error introduced by taking the linear mean—the value of α so derived being a slight underestimate. The magnitude of the systematic error due to this effect, which depends on the accuracy of determination of N_μ, can be as large as 0·1 in some cases.

There exists a further source of systematic error, which comes from inaccuracy in the determination of shower size N_e. Since the spectrum of detected showers is steep on both sides—at large sizes because of the steepness of the primary spectrum and at small sizes because of the fast decrease of the shower detection probability—the sizes of the largest detected showers will, in general, be overestimated and those of the smallest showers underestimated. The result is that the value of α is underestimated somewhat. The magnitude of the error depends strongly on the accuracy of size determination and the width of the size interval adopted. No corrections for this effect have been made in the present work because of lack of detailed knowledge of the conditions in the various experiments.

4.2. *Experimental measurements of α for showers of fixed size*

The available experimental data, from near-sea-level experiments, have been utilized in figure 3, which shows the variation of α with $\bar{N}_e$ for showers of fixed size. A key to the

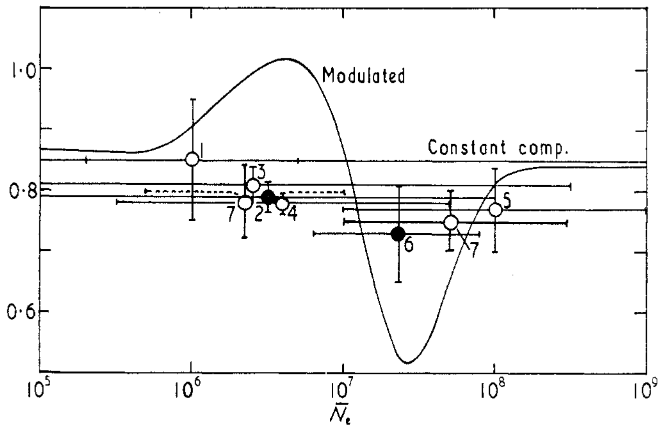

Figure 3. Variation of α, the coefficient in the expression $\bar{N}_\mu \propto \bar{N}_e{}^\alpha$, with shower size for near-vertical showers of constant electron size.

experimental values is given in table 1. Where the basic data were given by the authors, a re-analysis has been made by us, bearing in mind the factors mentioned in § 4.1.

Also shown in figure 3 are the theoretical predictions for two primary mass spectra—the modulated spectrum (see II for details) and a primary spectrum of constant composition. In view of the near constancy of α from one primary mass to another, the composition

Table 1. The source of the α values given in figures 3 and 5

References			
Fixed electron size		Fixed muon size	
1	Khrenov 1962	1	Vernov *et al.* 1967
2	Hasegawa *et al.* 1963	2	Hasegawa *et al.* 1963
3	Nikolski 1962		
4	Vernov *et al.* 1967		
5	Linsley and Scarsi 1962		
6	Pidcock 1967†		
7	Earnshaw *et al.* 1967 b		

† The values of α have been estimated by the present authors.

remaining constant, it is not necessary to specify what that composition is. If we consider first the overall average value of α, it can be seen that the experimental measurements indicate a mean close to 0·8, whereas the theoretical mean is a little higher than this. However, such a discrepancy cannot be regarded as meaningful in view of two factors:

(i) Small imperfections in the adopted model, such as the neglect of secondary particles other than pions and nucleons.

(ii) The existence of a residual bias in the experimental values (see § 4.1) which causes them to be underestimates.

Turning now to the search for a variation of α with $\bar{N}_e$, which would throw light on the problem of the mass composition, it is seen that in most cases the experimental data refer to an average over such a wide range of size that an oscillation of α, as would follow from a modulated spectrum, would be smoothed out. Exceptions occur in the measurements from the Haverah Park experiment reported by Pidcock (1967) and Earnshaw *et al.* (1967 b) which refer to comparatively narrow regions of size. The value indicated in figure 3 for the measurements of Pidcock has been calculated by us using the median muon number method and applying a correction to allow for the fact that, because the apparatus uses rather deep Čerenkov detectors, the contribution of the muon component to the measured size is unusually large. A consequence of this muon sensitivity is that the size determined directly from the data, the nominal size N_n, has to be related to the 'conventional' size N_c, as would be detected with a conventional detecting array, by a standardization procedure which compares the size spectra measured in the two cases. It has been shown (Suri 1966) that the relation between the two sizes is

$$N_n \propto N_c^{0\cdot 8} \tag{1}$$

and this relation has been used together with the N_μ, N_n values to give the value of α shown in figure 3.

Relation (1) itself can be used to give an almost independent value of α. It is estimated that half of the Čerenkov detector response is due to muons so that, assuming that the non-linearity expressed by relation (1) is due only to the decrease of the muon ratio with increasing shower size, the corresponding value of α is 0·74, with an estimated error of $\pm 0\cdot 05$. This value adds weight to the suggestion that the value of α is a little low in the region. Similarly, the results of Earnshaw *et al.* give a value of α which is a little low in the size region above 10^7, although the value shown has been taken from the published variation of $\bar{N}_\mu$ with $\bar{N}_e$ rather than from the basic data and no corrections have been applied for the various factors mentioned in § 4.1.

Some basic data have been given for the Tokyo experiment (Hasegawa *et al.* 1963) which are amenable to detailed study. Values of α have been calculated by the present authors for three cells of shower size with the result shown in figure 4. There appears to be some lack of constancy here, but the results do not follow the undulating curve. The first point is unusually low, and this is just in the region of small shower sizes where, in figure 1, very large fluctuations in shower size for fixed muon number were indicated. Both facts suggest rather large errors in size determination for these small showers.

The very slight evidence that exists, suggesting a change in composition, is the small reduction in α for the last point in figure 4 and the slightly lower value given in figure 3 for the Haverah Park experiment (Pidcock 1967), but clearly such evidence is far from definitive.

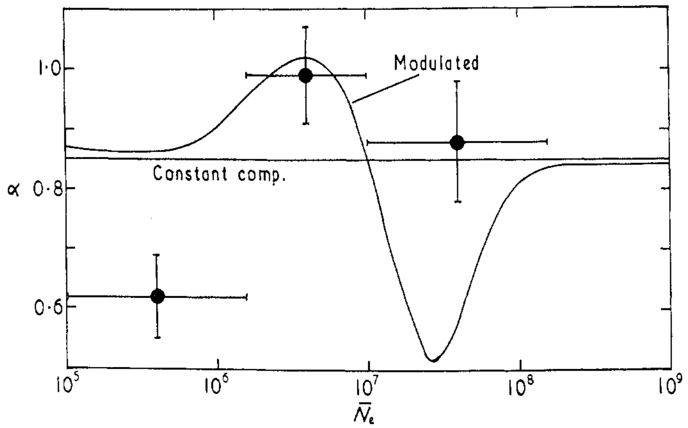

Figure 4. Variation of α with shower size for the near-vertical showers of constant size recorded in the experiment of Hasegawa *et al.* (1963).

4.3. *Experimental measurements of α for showers of fixed muon number*

A comparison is made in figure 5 between expectation and observation of α as a function of $\bar{N}_e$ in showers of fixed muon number. Unfortunately, the averages are given over very wide ranges again, so that no undulation can be expected, but the mean value is in the expected region.

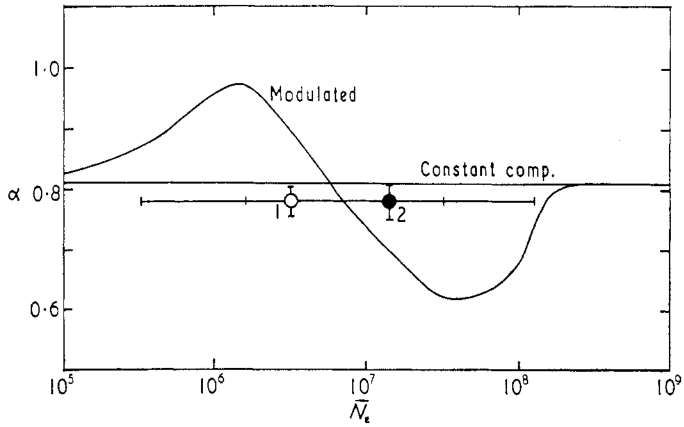

Figure 5. Variation of α with shower size for near-vertical showers of constant muon number.

5. Conclusions

The conclusions can be listed as follows:

(i) The predictions of the preferred model (using the CKP relation) with regard to the magnitude of the fluctuations in N_μ/N_e and the average (N_μ, N_e) dependence are in good agreement with the experimental data.

(ii) The accuracy of the experimental data on fluctuations is not yet sufficient to enable a conclusion to be drawn about the primary mass composition.

(iii) The exponent α in the relation $\bar{N}_\mu \propto \bar{N}_e{}^\alpha$ is subject to a variety of experimental biases which commonly lead to its underestimate.

(iv) The value of α is sensitive to the primary composition, but the existing experimental data are too poor to enable a firm conclusion to be drawn; what evidence there is gives only a very slight suggestion of some change in composition.

(v) There is a great need for a very careful experiment with large area detectors and an analysis in which allowance is made for the various bias effects. Such an experiment would throw much more light on the mass composition problem.

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